5–11 Using Several Methods of Factoring

Objective: To factor polynomials completely.

Vocabulary

Factored completely A polynomial expressed as the product of a monomial and one or more prime polynomials, that is when it cannot be factored further.

Guidelines for Factoring Completely

1. Factor out the greatest monomial factor first.

2. Look for a difference of squares.
   Pattern: $a^2 - b^2 = (a - b)(a + b)$ (However, $a^2 + b^2$ can't be factored.)

3. Look for a perfect square trinomial.
   Patterns: $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$

4. If the trinomial is not a perfect square, look for a pair of binomial factors.

5. If a polynomial has four or more terms, look for a way to group the terms in pairs or in a group of three terms that is a perfect square trinomial.

6. Make sure that each binomial or trinomial factor is prime.

7. Check your work by multiplying the factors.

Example 1   Factor $8x^3 - 512x$ completely.

Solution $8x^3 - 512x = 8x(x^2 - 64)$
             Greatest monomial factor ⎯⎯⎯↑   ↑⎯⎯⎯ Difference of squares
                           $= 8x(x + 8)(x - 8)$

Example 2   Factor $3x^3 + 3x^2 - 18x$ completely.

Solution $3x^3 + 3x^2 - 18x = 3x(x^2 + x - 6)$
             Greatest monomial factor ⎯⎯⎯↑   ↑⎯⎯⎯ Trinomial
                           $= 3x(x + 3)(x - 2)$

Factor completely.

1. $3x^3 - 12x$ **$3x(x + 2)(x - 2)$**

2. $5m^3 - 45m$ **$5m(m + 3)(m - 3)$**

3. $3a^2 + 6ab + 3b^2$ **$3(a + b)^2$**

4. $-x^3 + 4xy^2$ **$-x(x + 2y)(x - 2y)$**

5. $-12z^3 + 30z^2 + 18z$ **$-6z(2z + 1)(z - 3)$**

6. $16r^4 - 24r^3 + 9r^2$ **$r^2(4r - 3)^2$**

7. $20x^3 - 28x^2 + 8x$ **$4x(x - 1)(5x - 2)$**

8. $t^3 + t^2 - 2t$ **$t(t - 1)(t + 2)$**

9. $2x^2 - 128$ **$2(x + 8)(x - 8)$**

10. $2x^4 - 162$ **$2(x^2 + 9)(x + 3)(x - 3)$**

11. $25z^3 - 36y^2z$ **$z(5z + 6y)(5z - 6y)$**

12. $6x^2 + 22xy - 8y^2$ **$2(3x - y)(x + 4y)$**

5–11 Using Several Methods of Factoring (continued)

Example 3   Factor $5a^2b^3 + 2a^3b^2 - 3ab^4$ completely.

Solution   First rewrite the polynomial in order of decreasing degree in a.

$5a^2b^3 + 2a^3b^2 - 3ab^4 = 2a^3b^2 + 5a^2b^3 - 3ab^4$

$= ab^2(2a^2 + 5ab - 3b^2)$

Greatest monomial factor ⎦   Trinomial

$= ab^2(2a - b)(a + 3b)$

Example 4   Factor $a^2b - 4b + 3a^2 - 12$ completely.

Solution $a^2b - 4b + 3a^2 - 12 = b(a^2 - 4) + 3(a^2 - 4)$ Group and factor.

$= (b + 3)(a^2 - 4)$ Use the distributive property.

Difference of squares

$= (b + 3)(a + 2)(a - 2)$

Factor completely.

13. $a^3x - 9ax^3$ **$ax(a + 3x)(a - 3x)$**

14. $18x^3 - 24x^2 + 8x$ **$2x(3x - 2)^2$**

15. $20 - 60x + 45x^2$ **$5(3x - 2)^2$**

16. $6x^2 - 18xy + 12y^2$ **$6(x - y)(x - 2y)$**

17. $9x^3 + 108x + 63x^2$ **$9x(x + 3)(x + 4)$**

18. $10k^3 + 25k - 35k^2$ **$5k(2k - 5)(k - 1)$**

19. $32r^4 - 48r^3 + 18r^2$ **$2r^2(4r - 3)^2$**

20. $12ab - 3b^2 - 12a^2$ **$-3(2a - b)^2$**

21. $bc^2 - 4b + 3c^2 - 12$

22. $x^3 - x + 6x^2 - 6$ **$(x + 6)(x + 1)(x - 1)$**

23. $x^2 + 6xy + 9y^2 - 16$

24. $a^3 + a^2b - ab^2 - b^3$ **$(a + b)^2(a - b)$**

25. $y^4 - 9y^2 + 20$ **$(y^2 - 5)(y + 2)(y - 2)$**

26. $x^4 - 10x^2 + 9$ **$(x + 1)(x - 1)(x + 3)(x - 3)$**

27. $x^4 - 13x^2 + 36$

28. $x^4 - 24x^2 + 144$ **$(x^2 - 12)^2$**

29. $b^4 - 8b^2 + 16$ **$(b + 2)^2(b - 2)^2$**

30. $a^3 + 2a^2 - 5a - 10$ **$(a + 2)(a^2 - 5)$**

21. **$(b + 3)(c + 2)(c - 2)$**
23. **$(x + 3y + 4)(x + 3y - 4)$**
27. **$(x + 2)(x - 2)(x + 3)(x - 3)$**

Mixed Review Exercises

Simplify.

1. $\left(-\frac{1}{3}\right)\left(\frac{1}{4}\right)(60)$ **-5**

2. $\frac{1}{8}(56)$ **7**

3. $-\frac{1}{7}(56)\left(-\frac{1}{8}\right)$ **1**

4. $\frac{120b}{8}$ **$15b$**

5. $45 \div \left(\frac{1}{5}\right)$ **225**

6. $600 \div (-5)$ **-120**

Factor.

7. $x^2 - 11x + 30$ **$(x - 6)(x - 5)$**

8. $x^2 + 2x - 35$ **$(x + 7)(x - 5)$**

9. $x^2 - x - 20$ **$(x - 5)(x + 4)$**

10. $2n^2 + 15n + 7$ **$(2n + 1)(n + 7)$**

11. $3x^2 + 7x + 4$ **$(3x + 4)(x + 1)$**

12. $(2x - 6) - 3n(3 - x)$ **$(2 + 3n)(x - 3)$**